## Problem 5-1

In this problem, we have to determine a linear state-feedback controller such that the closed-loop poles match those given in the problem.

The system given in the problem is indicated in Equation (1-1) and Equation (1-2).

|  |  |
| --- | --- |
|  | (1-1) |
|  | (1-2) |

Since the closed-loop poles are situated at 0.1 and 0.25, the desired characteristic equation can be written out as in Equation (1-3). We then get Equation (1-4) by expanding Equation (1-3).

|  |  |
| --- | --- |
|  | (1-3) |
|  | (1-4) |

We then set the state-feedback controller  as in Equation (1-5).

|  |  |
| --- | --- |
|  | (1-5) |

For state-feedback control system, the closed-loop characteristic equation can be expressed as of line two on page 14 in [1: Lian 2019], which is also shown here in Equation (1-6). By modifying Equation (1-6) and adopting the Eigenvalue Assignment as mentioned on page 14 and 24 in [1: Lian 2019], we obtain the closed-loop characteristic equation of the system as in Equation (1-7).

|  |  |
| --- | --- |
|  | (1-6) |
|  | (1-7) |

According to page 11 in [1: Lian 2019], we can see that the discrete time model with given h can be expressed as in Equation (1-8). By comparing Equation (1-8) with the given system as indicated in Equation (1-1) and Equation (1-2), we can set matrices F and H as in Equation (1-9) and Equation (1-10) respectively.

|  |  |
| --- | --- |
|  | (1-8) |
|  | (1-9) |
|  | (1-10) |

By substituting the values of matrices F and H from Equation (1-9) and Equation (1-10) into Equation (1-7), Equation (1-7) can then be rewritten as Equation (1-11).

|  |  |
| --- | --- |
|  | (1-11) |

By expanding Equation (1-11), we get Equation (1-12).

|  |  |
| --- | --- |
|  | (1-12) |

As Equation (1-12) should be equivalent to the desired characteristic equation as in Equation (1-4), we get the following Equation (1-13) and Equation (1-14).

|  |  |
| --- | --- |
|  | (1-13) |
|  | (1-14) |

By solving Equation (1-13) and Equation (1-14), we obtain the values of  and  as in Equation (1-15) and Equation (1-16).

|  |  |
| --- | --- |
|  | (1-15) |
|  | (1-16) |

The linear state-feedback controller can thus be expressed as in Equation (1-17), which is also the solution for the problem.

|  |  |
| --- | --- |
|  | Ans.  (1-17) |

## Problem 5-2

Given equations for both a continuous-time system and a sampled system with  set as 0.2, the problem asks us to determine a linear state-feedback controller such that it satisfies a specific closed-loop characteristic polynomial. A simulation on the closed-loop system with a given initial state should also be performed.

The continuous-time system given in the problem is indicated in Equation (2-1) and Equation (2-2), while the sampled system is shown in Equation (2-3).

|  |  |
| --- | --- |
|  | (2-1) |
|  | (2-2) |
|  | (2-3) |

Part (a):

In part (a), we have to determine a linear state-feedback controller. Similar to Problem 5-1, we adopt the eigenvalue assignment and modify the closed-loop characteristic equation as of line two on page 14 in [1: Lian 2019], which is also shown here in Equation (2-4). The modified equation is indicated in Equation (2-5) .

|  |  |
| --- | --- |
|  | (2-4) |
|  | (2-5) |

According to page 11 in [1: Lian 2019], we can see that the discrete time model with given h can be expressed as in Equation (2-6). By comparing Equation (2-6) with the given system as indicated in Equation (2-3), we can set matrices F and H as in Equation (2-7) and Equation (2-8) respectively.

|  |  |
| --- | --- |
|  | (2-6) |
|  | (2-7) |
|  | (2-8) |

We then set the linear state-feedback controller  as in Equation (2-9).

|  |  |
| --- | --- |
|  | (2-9) |

By substituting the values of matrices F and H from Equation (2-7) and Equation (2-8) into Equation (2-5), Equation (2-5) can then be rewritten as Equation (2-10).

|  |  |
| --- | --- |
|  | (2-10) |

By expanding Equation (2-10), we get Equation (2-11).

|  |  |
| --- | --- |
|  | (2-11) |

As Equation (2-11) should be equivalent to the desired characteristic equation as shown in Equation (2-12), which is provided by the problem, we get the following Equation (2-13) and Equation (2-14).

|  |  |
| --- | --- |
|  | (2-12) |
|  | (2-13) |
|  | (2-14) |

By solving Equation (2-13) and Equation (2-14), we obtain the values of  and  as in Equation (2-15) and Equation (2-16).

|  |  |
| --- | --- |
|  | (2-15) |
|  | (2-16) |

The linear state-feedback controller can thus be expressed as in Equation (2-17), which is also the solution for part (a) of Problem 5-2.

|  |  |
| --- | --- |
|  | Ans.  (2-17) |

Part (b):

In part (b), we wish to simulate the closed-loop system at . The state input and output of the systems are shown in the following figures. The plant model used in the simulation is the continuous-time model indicated in Equation (2-1) and Equation (2-2).

All simulations are performed using Matlab. To simulate the initial condition response of a state-space model, we use the syntax and the example code mentioned in [2], which refers to command *initial*. To convert the model from continuous time to discrete time, the syntax in [3] is used. Command *ssdata* is used to access state-space model data in the output simulation. For the matrix parameters used in the simulations, matrix H and C are as indicated in Equation (2-18) and Equation (2-19).

|  |  |
| --- | --- |
|  | (2-18) |
|  | (2-19) |

The matrix F for simulating the input is defined as in Equation (2-20) while the F for the output simulation, which is defined as in Equation (2-21), takes the linear state-feedback controller into account.

|  |  |
| --- | --- |
|  | (2-20) |
|  | (2-21) |

As for the output, the system is discretized, hence we use the command *stairs* instead of *plot* to sketch the result. The result for the input and the output signals are shown in Figure 2-1 and Figure 2-2. Figure 2-3 shows the states of the closed-loop system.

|  |
| --- |
|  |
| Figure 2-1. The input signal for the system given in Problem 5-2. |

|  |
| --- |
|  |
| Figure 2-2. The output signal for the system given in Problem 5-2. |
|  |
| Figure 2-3. The states for the closed-loop system. The upper line indicates the states for the output, while the bottom line indicates the states for the input. |

## Problem 5-3

In this problem, we have to determine a linear state-feedback controller and an observer such that the states are brought to the origin in two sampling intervals and the observer has the desired characteristic polynomial  for a given system. We also have to determine whether a linear state-feedback controller is capable of taking the system from the origin to .

Part(a):

For part(a), we have to determine a linear state-feedback controller as given in Equation (3-1) such that the states can be brought to the origin in two sampling intervals.

|  |  |
| --- | --- |
|  | (3-1) |

According to page 11 in [1: Lian 2019], we can see that the discrete time model with given h can be expressed as in Equation (3-2). By comparing Equation (3-2) with the given system as indicated in Equation (3-3) and Equation (3-4), we can set matrices F and H as in Equation (3-5) and Equation (3-6) respectively.

|  |  |
| --- | --- |
|  | (3-2) |
|  | (3-3) |
|  | (3-4) |
|  | (3-5) |
|  | (3-6) |

For state-feedback control system, the closed-loop characteristic equation can be expressed as of line two on page 14 in [1: Lian 2019], which is written out here as in Equation (3-7). By substituting the values of matrices F and H from Equation (3-5) and Equation (3-6) into the left side of Equation (3-7), Equation (3-7) can then be rewritten as Equation (3-8).

|  |  |
| --- | --- |
|  | (3-7) |
|  | (3-8) |

According to the linear state-feedback controller as indicated in Equation (3-1), the  in Equation (3-8) can therefore be expressed as in Equation (3-9). Now that we have the values for matrices F, H, and K, the determinant in Equation (3-8) can then be rewritten into Equation (3-10).

|  |  |
| --- | --- |
|  | (3-9) |
|  | (3-10) |

By computing the determinant in Equation (3-10), we obtain the characteristic equation as indicated in Equation (3-11).

|  |  |
| --- | --- |
|  | (3-11) |

Since the states of the linear state-feedback controller are brought to the origin in two sampling intervals as mentioned in the problem, Equation (3-11) should therefore be equivalent to , which results in Equation (3-12).

|  |  |
| --- | --- |
|  | (3-12) |

It is easily seen that the solution for Equation (3-12) is not unique. We choose the set in Equation (3-13) as the solution.

|  |  |
| --- | --- |
|  | Ans.  (3-13) |

Part (b):

To know if whether the state-feedback controller can take the system from the origin to , we have to compute the controllability matrix to determine reachability. From page 20 in [1: Lian 2019], we can express the controllability matrix  as in Equation (3-14). By applying the F and H values as indicated in Equation (3-5) and Equation (3-6) into Equation (3-14), we can compute the controllability matrix as in Equation (3-15).

|  |  |
| --- | --- |
|  | (3-14) |
|  | (3-15) |

By computing the determinant of , we find that the determinant of the controllability matrix is zero, which indicates that the system is not reachable. In other words, it is impossible for any state to be reached from the origin. However, the given state in the system, , belongs to the column space of the controllability matrix. This makes it possible for the state to be reached from the origin.

According to the system model on page 12 in [1: Lian 2019], we can obtain Equation (3-16) and Equation (3-17). And by substituting Equation (3-16) into Equation (3-17), we can obtain Equation (3-18). We then get Equation (3-19) by expanding Equation (3-18).

|  |  |
| --- | --- |
|  | (3-16) |
|  | (3-17) |
|  | (3-18) |
|  | (3-19) |

While  is given in the problem as , the initial state is set as . By applying the values of  and  into Equation (3-19), we get the following Equation (3-20) and Equation (3-21).

|  |  |
| --- | --- |
|  | (3-20) |
|  | (3-21) |

By solving Equation (3-20) and Equation (3-21), we can see that there is no unique solution. We can choose one set as mentioned in Equation (3-22) and Equation (3-23).

|  |  |
| --- | --- |
|  | (3-22) |
|  | (3-23) |

Since we have obtained a set of solution, it is therefore possible to determine a linear state-feedback controller that can take the system from the origin to .

Part (c):

Given that the desired characteristic polynomial for the observer is , we can expand the given characteristic equation of the observer into Equation (3-24).

|  |  |
| --- | --- |
|  | (3-24) |

The closed-loop characteristic equation for the observer can be expressed as the left side of Equation (3-25), where the matrix C, which is shown in Equation (3-26), is defined according to the given system in Equation (3-4). Observer  is defined as in Equation (3-27) .

|  |  |
| --- | --- |
|  | (3-25) |
|  | (3-26) |
|  | (3-27) |

We then obtain Equation (3-28) by expanding and computing the determinant of the right side of Equation (3-25).

|  |  |
| --- | --- |
|  | (3-28) |

Since Equation (3-24) and Equation (3-28) are equivalent, we can get the following Equation (3-29) and Equation (3-30).

|  |  |
| --- | --- |
|  | (3-29) |
|  | (3-30) |

We then obtain the solution for the observer by solving Equation (3-29) and Equation (3-30). The solution is indicated in Equation (3-31).

|  |  |
| --- | --- |
|  | Ans.  (3-31) |

References

[1: Lian 2019]

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